International Conference on

EXPLORING THE HISTORY

OF

INDIAN MATHEMATICS

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IIT GANDHINAGAR

International Conference on

Exploring the History of Indian Mathematics

List of Speakers

P.P. DIVAKARAN; IUCAA, Pune & CMI, Chennai, INDIA

KIM PLOFKER; Union College, Schenectady, NY 12308, USA

AMARTYA KUMAR DUTTA, Indian Statistical Institute, Kolkata, INDIA

M.S. SRIRAM; Prof. K.V. Sarma Research Foundation, Adyar, Chennai, INDIA

PARTHASARATHI MUKHOPADHYAY; Department of Mathematics Ramakrishna Mission Residential College (Autn.), Narendrapur, Kolkata, INDIA

K. RAMASUBRAMANIAN; IIT Bombay, INDIA

M.D.SRINIVAS; Centre for Policy Studies, Chennai, INDIA

VENKETESWARA PAI R; Department of Humanities and Social Sciences, Indian Institute of Science Education and Research (IISER), Pune, INDIA

S.G. DANI; UM-DAE Centre for Excellence in Basic Sciences (CBS), Mumbai 400032, INDIA

CLEMENCY MONTELLE; University of Canterbury, NEW ZEALAND

ATUL DIXIT; IIT Gandhinagar, INDIA

MICHEL DANINO; IIT Gandhinagar, INDIA

ANIL NARAYANAN, N.; Chinmaya Sanskrit University, Kerala,INDIA

ADITYA KOLACHANA; IIT Bombay

ARADHANA ANAND; Delhi Public School, Bangalore South, INDIA

K. MAHESH; IIT Bombay

R. GONDHALEKAR; VPM's Academy of International Education and Research, Thane, INDIA

JAMBUGAHAPITIYE DHAMMALOKA; University of Canterbury, NEW ZEALAND & Department of Classical Languages, University of Peradeniya, Sri Lanka.

PRIYAMVADA, N.; Guruvayurappan Temple in Morganville, New Jersey, USA

SIVANANDAN D.S.; Centre for Spiritual Studies, Amritapuri Campus, Amrita University, INDIA

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Title and abstract

Invited Talk 1.

Title: Two masters: Aryabhata and Madhava, a reassessment

Speaker: P.P. DIVAKARAN; IUCAA, Pune & CMI, Chennai, INDIA

Abstract: There are, still, fresh insights to be gained from a study of these two masters, relating both to their mathematics – the sources of their creative ideas and the legacies they left behind – and their personal histories. There are also myths that have accumulated around their life and work which are no more than myths. Part of the talk will look at their (or their ancestors') journeys as a model of how Indian mathematics acquired its Indian identity.

Invited Talk 2.

Title: The language of heliocentrism in 18th-century Sanskrit mathematical astronomy

Speaker: KIM PLOFKER; Union College, Schenectady, NY 12308, USA

Abstract: The meaning and application of Sanskrit scientific technical terms were scrupulously evaluated by commentators throughout the long history of the technical genre of *jyotisa* or mathematical and astral sciences. This paper examines how Sanskrit technical vocabulary in the mathematical astronomy of early modern north India adapted to accommodate new cosmological concepts such as the heliocentric solar system, and how commentarial practices responded to such changes.

Invited Talk 3.

Title: Antiquity of the Arithmetic Mean

Speaker: AMARTYA KUMAR DUTTA, Indian Statistical Institute, Kolkata, INDIA

Abstract: The importance of the Arithmetic Mean was articulated in 1809 by C.F. Gauss, one of the pioneers of mathematical statistics, as follows: "it has been customary certainly to regard as an axiom the hypothesis that if any quantity has been determined by several direct observations, made under the same circumstances and with equal care, the arithmetic mean of the observed values affords the most probable value, if not rigorously, yet very nearly at least, so that it is always safe to adhere to it."

Like the decimal system, the Arithmetic Mean now appears as a natural concept to us who have grown up with it. But the history of the concept shows that its introduction must have involved conceptual subtleties. S.M. Stigler, a distinguished statistician of our time, considers the Arithmetic Mean as the first of the "five ideas that changed statistics and continue to change the way we think about the world".

The Arithmetic Mean has been variously perceived: as an exact mathematical concept, as an applied tool for combining measurements in experimental sciences, as a measure of the central tendency in statistical data, etc. Any measure of the central tendency seeks to identify that [central] value of the distribution which can be taken to be its best representative. In this talk, we shall discuss the emergence of some of the avatars of the Arithmetic Mean at different time points, in different contexts, in different mathematics cultures.

The word "mean" is derived from Old French *meien* meaning "middle" or "centre". And indeed the Arithmetic Mean captures the concept of the "centre". In fact, the first recorded definition of Arithmetic Mean is as a geometry concept in ancient Greece (c. 500 BCE): a number equidistant from two given numbers, i.e., as the "middle" point of two numbers. But it does not appear to have been envisaged as a statistics concept (average or best representative).

The general Arithmetic Mean $\overline{x} = \frac{x_1+x_2+\cdots+x_n}{n}$ and its statistical applications have not been found in European treatises prior to the sixteenth century CE. The formulation $\overline{x} = \frac{x_1+x_2+\cdots+x_n}{n}$ (whether expressed through symbols or words) is something intrinsically algebraic and, for effective application, requires good algorithms for performing all the elementary arithmetic operations. The appearance of the arithmetic mean in Europe coincides with her adoption of the decimal system and arithmetic methods based on the decimal system (which are, in essence, of Indian origin) and with the emergence of algebra.

A basic idea at the heart of the Arithmetic Mean as a statistical estimate is to combine observations — to replace several numbers by a single number. Thinkers on statistics like Stigler feel that this idea is counterintuitive: for it seeks, paradoxically, to gain information about the data by discarding information, namely, the individuality of the observations. This could also explain the delay among European scientists in developing a precise method (like Arithmetic Mean) for obtaining a best estimate on the basis of several observations. They relied on their judgements to select a particular observation thought to be the best rather than combine (and thereby spoil) it with other observations.

The technique of repeating and combining measurements on the same quantity can be seen in the work of Tycho Brahe and a few other astronomers towards the end of the sixteenth century, but the method for combining the repeated observations into a single number is not stated explicitly. Historians of statistics have usually identified the earliest unambiguous statistical use of the Arithmetic Mean in a work of the English astronomer Henry Gellibrand (1635). In his Presidential address at the American Statistical Association in 1971, Churchill Eisenhart stated, "I fully expected that I would find some good examples of mean-taking in ancient astronomy; and, perhaps, also in ancient physics. I have not found any. And I now believe that no such examples will be found in ancient science."

But, as in the case of numerous other scientific concepts, the above account of history completely overlooks the clear, precise and abundant use of the Arithmetic Mean in the treatises of ancient Indian mathematicians like Brahmagupta (628 CE), Śrīdharācārya (c. 750 CE), Mahāvīrācārya (850 CE), Pṛthūdakasvāmī (864 CE), Bhāskarācārya (1150 CE) and others. In fact, the Indian mathematicians define and apply the more general and sophisticated concept of the *weighted* Arithmetic Mean. An intuitive awareness of the law of large numbers for Arithmetic Mean also comes out in a commentary by the mathematician Gaṇeṣa (1545 CE).

The chapters in Indian arithmetic called *khāta-vyavahāra* (mathematical processes pertaining to excavations) describe how to compute the [average] depth, width or length of an irregular-shaped pool of water and thereby estimate its volume. It is in these chapters that the Arithmetic Mean is defined and used in the statistical sense — as the best representative value for a set of observations. Brahmagupta is one of the earliest Indian mathematicians in whose text the concept of [weighted] Arithmetic Mean occurs explicitly in this sense. He uses it to represent the depth of a ditch, when the depth is different in different portions of the ditch. The ancient Indian terms for Arithmetic Mean, like *samarajju* (mean measure of a line segment) or *samamiti* (mean measure) which use the word *sama* (equal, equable, same, common, mean) confirm that the Arithmetic Mean was perceived by ancient Indian scholars as the "common" or "equalizing" value which would be the appropriate representative measure for various observed measurements. The treatment of the excavation problems through the Arithmetic Mean was possibly one of the factors which shaped the gradual development of calculus in India.

The weighted arithmetic mean also appears in ancient Indian arithmetic treatises as an exact mathematical formula in chapters on *miśraka-vyavahāra* (computations pertaining to mixtures) which address the problem of computing the proportion of pure gold in an alloy formed by blending of several pieces of gold of different weights and purities.

Ancient India had a strong tradition in computational mathematics and algebra which provided a conducive mathematical environment for the emergence of the statistical arithmetic mean. The conceptualization of the Arithmetic Mean involves the idea of *combining* several numbers into a single number. Ancient Indian mathematics is replete with various ideas of "combination". One is reminded of the profound *bhāvanā* of Brahmagupta — a law of composition which *combines* two solutions of a certain quadratic equation in three variables to produce another solution of the equation, a principle with momentous consequences in mathematics.

The talk is a tribute to Prasanta Chandra Mahalanobis (1893–1972), the pioneer in research and use of statistics in modern India and the founder of the Indian Statistical Institute, on his 125th birth anniversary.

Invited Talk 4.

Title: Spherical trigonometry in medieval Indian texts on astronomy

Speaker: M.S. SRIRAM; Prof. K.V. Sarma Research Foundation, Adyar, Chennai, INDIA

Abstract: For solving problems in positional astronomy, we need to know the properties of triangles drawn on spherical surfaces. This is the subject matter of spherical trigonometry. Consider a spherical triangle with sides a,b and c, and the spherical angles, A,B and C. Then, the fundamental formulae for spherical triangles are:

$$\text{Cosine formula}: \quad \cos a = \cos b \cos c + \sin b \sin c \cos A,$$

$$\text{and, Sine formula}: \quad \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}.$$

All the possible relations among the arcs and angles between them in a spherical triangle can be derived from these two formulae (example: Four parts formula).

How did the Indian astronomy texts handle the relations among the arcs and the angles between them, which are needed for solving the spherical astronomy problems, including the diurnal motion of the celestial objects, eclipses, and so on? Some of the problems can be solved by reducing them to ones of plane trigonometry, using some careful geometrical constructions. This is especially true of Bhāskaras ingenious solutions of the diurnal problems in the chapter on *Triprasnādhikāra* in *Siddhāntasiromaṇi* (1150 CE).

There is a significant progress of the methods in the two Kerala works, Tantrasangraha (1500 CE) and Yuktibhāsā (approx. 1530 CE). Consider the 5 variables, zenith distance (z), azimuth (a), hour angle (H), declination (δ), and the terestrial latitude (ϕ). Given any three of them, the other two can be determined. So, there are $\binom{5}{2} = 10$ problems. *Tantrasaṅgraha* deals with them systematically and gives the solutions. *Yuktibhāṣā* provides the complete derivation of these. Some of them involve the repeated application of the 'declination-type

formula only[1], and not reduction of the problems to ones of plane trigonometry. It appears that *Karaṇapaddhati* of Putumana Somayājī (probably 16th century) uses the *Yuktibhāṣā* methods to solve some fairly non-trivial problems.

Invited Talk 5.

Title: ZERO - An Eternal Enigma

Speaker: PARTHASARATHI MUKHOPADHYAY; Department of Mathematics Ramakrishna Mission Residential College (Autn.), Narendrapur, Kolkata, INDIA

Abstract: Historical reconstruction of mathematical knowledge, that was likely to be prevalent in Indian Antiquity or at an earlier period, is much like arranging an enormous jigsaw puzzle, many pieces of which are missing. Competent historians of Mathematics, all over the world, are trying to arrange the available pieces, according to their own respective 'stance's, with an obvious intention to try and guess the picture that it may suggest. And the job is anything but easy. Patriotic passion or pre-conceived ideas sometime come in the way of scholarly acumen, prompting one to misplace one or two pieces or perhaps not to place them at all, distorting the figure to accommodate one's 'stance'. Of course, there are scholars, carefully steering a middle course, analyzing objectively as far as possible, the available pieces of information or sometimes even the lack of information, trying to make a pattern out of it. With every new piece being found occasionally, as a new input to the jigsaw puzzle, one has to try and find its right place in the puzzle, sometimes destroying the existing pattern. And the journey continues.

The genesis of 'zero' as a number, that even a child so casually uses today, is a long and involved one. As a passionate observer with keen interest to know the actual truth about its origin, from a vast literature of scholarly articles, one finds that a substantial number of persons concerned with the history of its evolution, today believe that the number 'zero' - in its true potential, as we use it in our present day mathematics - has its root, conceptually as well as etymologically, in the Sanskrit word 'Sūnya' of Indian Antiquity. However, its exact time of origin is still hotly debated, a recent controversy in this direction being the outcome of carbon dating of the famous *Bakshali* manuscript. Going in other direction, some scholars suggest that a trace of this concept, if not in total operational perspective, might have a Greek origin that traveled to India during the Greek invasion of the northern part of the country in the pre-Mauryan period. A relatively recent third view professes the Chinese origin of the concept of zero as a number, that might have traveled through the traders from China to the far east part of Asia, to places like Cambodia, then under Buddhist influence that spread from

[1]This is the formula for the distance of a point on an arc from another arc, to which it is inclined.

mainland India, where it got the shape of 'bold dot', the earliest known written form of zero. They credit the mainland India only for 'garlanding' this symbol of zero towards its modern shape, sometime around the 8th or 9th century C.E.

We, the average educated Indians seem to be quite proud of the fact that in some remote past, Indians discovered the number zero. But neither was it invented in a laboratory, nor excavated from somewhere! What really is meant by this discovery then? To know this answer from the proper perspective, one has to scan through a rich history of early civilizations through a period of over 5000 years. In this engaging tour of history, the journey takes us through the Egyptian Hieroglyphics and Inca Quipu, via Babylonian clay tablets and Mayan Glyphs, followed by the great Greek civilization and the mighty Roman empire in the West, whereas we come across the lofty philosophy of ancient Indian seers and industrious pursuits of wise Arabians of Baghdad in the East. Touching these cornerstones, this lecture will try to place the true genesis of our modern zero in its proper historical footing - an ode to 'The Nothing - That Is' !

Invited Talk 6.

Title: From short Sūtras to melodious meters to paragon of poetry: Evolution of the style of composition in Indian Mathematics

Speaker: K. RAMASUBRAMANIAN; IIT Bombay, INDIA

Abstract: Starting from the most elementary thing in mathematics namely the representation of numbers, through the way of expressing recursive relations, to arriving at the solutions of indeterminate equations, to the development of sophisticated techniques in handling the infinite and the infititesimals, there has been a wide variation in the working-style of mathematicians of different cultures. The word working-style is employed here in a broad sense to inlcude the mathematicians's approach (i) in formulating a given problem, (ii) in visualizing the solution, (iii) in representing them in a tangible form using a local language (including mnemonics, symbols, etc), (iv) in defining the purpose of mathematics, and the like.

If we look at the corpus of knowledge that has been produced in India in ancient times—be it in mathematics, logic, prosody, philosophy or grammar—all of them have been primarily composed in the form of *sūtras*. A *sūtra*, by definition, is supposed to be composed in such a way, that is quite short, brief and to the point without any exegesis, unless it is warranted to be long. However, starting from the 5th century CE there seems to be a change in trend in the style of composition. Most of the primary works that were composed in mathematics or astronomy have been in the form of metrical compositions. The shift to compose in the form

of poetry can be easily understood given the fact that this style would greatly facilitate the transmission and preservation of knowledge that was primarily oral.

The Indian mathematicians quickly became so adept in adopting this style, and found several ingenious ways of expressing numbers that would satisfy the metrical constraints. The advent of various mathematical ideas and concepts, also compelled them to develop new terminologies to refer to them and the mathematical processes. The Sanksrit language due it is rich vocabulary as well as compound structure, enabled them to express even infinite series expansions of π and other trigonometrical functions in the form of beautiful verses, sometimes even with a *double entendre*.

Unfortunately, due to lack of proper grounding in the tradition, clubbed with inappropriate appraisals that were made in the early stages (not to mention biases), made historians of mathematics declare the approach of Indian mathematicians as mere *tricks*. During our presentation, we will show why such conclusions are ill-founded, and purely stem from misunderstanding of the cultural background peculiar to India.

Invited Talk 7.

Title: The *Chakravala* method compared with the Euler-Lagrange method for solving the Quadratic Indeterminate Equation $x^2 - Dy^2 = 1$

Speaker: M.D.SRINIVAS; Centre for Policy Studies, Chennai, INDIA

Abstract: Solutions of indeterminate equations have been considered in the Indian mathematical tradition from the time of the *Sulvasutras* (prior to 800 BCE). In his *Aryabhatiya* (c. 499 CE), Aryabhata presented the *Kuttaka* method for the solving the linear indeterminate equation $ax - by = c$, where a, b and c are given integers, and the problem is to find integral solutions for x and y. The method involves the mutual division of a and b, and is similar to the computation of the convergents in the simple continued fraction expansion of the rational number $\frac{a}{b}$.

In his *Brahmasphutasiddhanta* (c.628 CE), Brahmagupta considered the *Vargaprakriti* equation $x^2 - Dy^2 = k$, where D, k are given integers (D, the *prakrti*, is a non-square integer, and k is called the $kshepa$), and the problem is to find integral solutions for x, y. This problem seems to be closely related to the problem of finding rational approximations to square-roots which has been of interest to Indian mathematicians since the time of *Sulvasutras*. Brahmagupta discovered the important *bhavana* property, which enabled him to find solutions for the *kshepa* k_1k_2 once solutions are known for *kshepas* k_1 and k_2. Brahmagupta used the *bhavana* property also to solve the equation $x^2 - Dy^2 = 1$, for the particular cases $D = 83, 92$.

A general procedure for solving the equation $x^2 - Dy^2 = 1$, referred to as the *Chakravala* or the cyclic method, is presented by Bhaskaracharya in his celebrated text on algebra *Bijaganita* (c.1150). An earlier version of this method due to Jayadeva has been cited by Udayadivakara in a commentary written in 1073. A slightly different version of this method is discussed by Narayanapandita in his *Ganitakaumudi* (c.1356).

Indeterminate equations have been wrongly referred to as Diophantine equations, since the available portions of the *Arithmetica* of the Greek mathematician, Diophantus (c.250 CE) of Alexandria, deal only with rational and not integral solutions of equations. The Indian *kuttaka* method of solving linear indeterminate equations seems to have made its way to medieval Europe as it is found in a well known treatise on arithmetic writt en by Jordanus de Nemore in the 13th century. However, the French scholar Bachet de Mezeriac (1581-1638) presented the same as his own discovery in a book written in 1624. It was Bachets translation of the *Arithmetica* of Diophantus, that served as the source of inspiration for the investigations of Pierre de Fermat (1607-1653), the father of modern number theory.

In 1657, Fermat posed the problem of finding integral solutions of the equation $x^2 - Dy^2 = 1$, as a challenge to other French and British mathematicians. Incidentally, one of the specific cases mentioned by Fermat, namely $D = 61$, happens to be a problem whose solution has been worked out in the *Bijaganita* of Bhaskaracharya. The British mathematicians, William Brouncker (1620-1684) and John Wallis (1616-1703), came up with a method for solving all the specific cases mentioned by Fermat. In the eighteenth century, Leonhard Euler (1707-1783) made a systematic study of the equation x2-Dy2 = 1, though he wrongly named it after the British mathematician John Pell (1611-1685). Euler rediscovered the *bhavana* property, and also came up with the method of solving the equation by expressing $\sqrt{D}$ as a simple continued fraction. That this method always leads to a solution of the equation was proved by Joseph Louis Lagrange (1736-1813) in a series of papers published during 1768-1770.

In the early decades of 19th century, modern European scholars became acquainted with the *Chakravala* method and were duly impressed by the advances made in the subject by Indian algebraists centuries ago. However, it was generally presumed that the *Chakravala* method was only an earlier version of the more systematic Euler-Lagrange method. This misconception continues to be widely prevalent, even though in a series of seminal papers, published during 1929-1941, A. A. Krishnaswami Ayyangar (1892-1953) has shown that the *Chakravala* method was indeed different from the Euler-Lagrange method and is related to what is known as a semi-regular continued fraction expansion of $\sqrt{D}$. Ayyangar has also recast the *Chakravala* algorithm in a simpler form and shown that it always leads to the minimal or fundamental solution of the equation.

The ”Pell’s Equation” continues to be an important topic of investigation in modern mathematics. It is now known that the *Chakravala* method is more optimal than the Euler-Lagrange method: it takes considerably less number of steps (about 30% less than the Euler-Lagrange method on the average) in order to reach the solution.

Invited Talk 8.

Title: Mathematical basis for the $V\bar{a}kya$ system in $Karaṇapaddhati$

Speaker: VENKETESWARA PAI R; Department of Humanities and Social Sciences, Indian Institute of Science Education and Research (IISER), Pune, INDIA

Abstract: The term *vākya* literally means a sentence consisting of one or more words. In the context of astronomy, it refers to a phrase or a string of letters in which numerical values associated with various astronomical variables are encoded. The strings used in composing the *vākya* are chosen so that they not only represent numerical values, but are also in the form of beautiful meaningful phrases and sentences that convey worldly wisdom and moral values. The earliest set of *vākya*s known as *candra-vākya*s can be traced back to the 4th century and are attributed to Vararuci. From these one can obtain Moon’s true longitude for any desired day. This is described in the text *Vākyakaraṇa* (13th century) in which the fully developed *vākya* system is presented.

Vākyakaraṇa and other *vākya* texts only present the lists of *vākya*s and the simple arithmetical operations for obtaining the longitudes of the planets for any desired day. It is indeed the Karaṇapaddhati (16th century) of Putumana Somayājī which explains the mathematical basis of the *vākya* system. Karaṇapaddhati describes a mathematical technique known as *vallyupasaṃhāra* which is a variant of *kuṭṭaka* method for solving linear indeterminate equations. This is used for obtaining the smaller multipliers and divisors for a ratio which represents the rate of motion of the planets etc. *Vallyupasaṃhāra* method of transforming the *vallī* is essentially the recursive process of calculating the successive convergents of the continued fraction associated with the ratio.

In *Karaṇapaddhati*, Putumana Somayājī displays a very good understanding of the mathematical properties of the continued fraction expansion of a ratio of two integers G, H. Usually, G is the *guṇa* or *guakāra* or multiplier and H is the *hāra* or *hāraka* or divisor, and their ratio is the rate of motion of a particular planet or its apogee or node etc. Thus, G being the corrected revolution number and H the total number of civil days, they are indeed very large numbers. In this talk, we would present the algorithm and rationale for obtaining an integer known as *khaṇḍa* which is used in the computation of longitude of Moon using *candra-vākyas*. *Khaṇḍa* (in days, also known as *Khaṇḍadina*) is an auxillary epoch close to

ahargaṇa (number of civil days elapsed since the beginning of the epoch). On the day of *khaṇḍa*, Moon's anomaly would be close to zero at the sunrise. The rationale for obtaining *khaṇḍa* for any desired *ahargaṇa* is indeed an interesting mathematical problem.

Invited Talk 9.

Title: Syamadas Mukhopadhyaya : his life and works

Speaker: S.G. DANI; UM-DAE Centre for Excellence in Basic Sciences (CBS), Mumbai 400032, INDIA

Abstract: Syamadas Mukhopadhyaya (18661937) was an eminent geometer who worked at the University of Calcutta in the early decades of the twentieth century. He is renowned for his work on what is known as the four-vertex theorem in global differential geometry, and also in the area of hyperbolic geometry, which was a novel topic for the period. In this talk I shall trace his life and works.

Invited Talk 10.

Title: Computing Sines: Trigonometry in Nitynanda's Sarvasiddhntarja

Speaker: CLEMENCY MONTELLE; University of Canterbury, NEW ZEALAND

Abstract: The Sarvasiddhntarja ('King of all siddhntas') is a comprehesive work in Sanskrit on various aspects of astronomy which was composed in 1639 by Nitynanda, astronomer at the Mughal court of Shh Jahn in Delhi. Towards the beginning of the work, Nitynanda includes a long passage on trigonometry, since it is an indispensible tool for the astronomer. As well as including many conventional derivations of various trigonometric relations, Nitynanda includes certain novel features, some of which are his own insights and others which appear to be inspired by Arabic sources. I will explore a range of these to shed light on the character, scope, and inspiration of this passage.

Invited Talk 11.

Title: Ramanujan's formula for $\zeta(2m+1)$ and subsequent developments

Speaker: ATUL DIXIT; IIT Gandhinagar, INDIA

Abstract: While the Riemann zeta function $\zeta(s)$ at even positive integers is known to be always transcendental, the arithmetic nature of $\zeta(s)$ at odd positive integers remains mysterious with only $\zeta(3)$ known to be irrational, thanks to Apéry. In 1901, Lerch obtained a

beautiful result involving $\zeta(2m+1), m$ odd, and an Eisenstein series, which implies that either $\zeta(2m+1), m$ odd, or the Eisenstein series is transcendental. In his famous notebooks, Srinivasa Ramanujan obtained a beautiful result generalizing that of Lerch. This result has had tremendous impact on Mathematics with its applications ranging from modular forms to analysis of special data structures and algorithms. In the first part of the talk, we will give a historical survey on Ramanujan's formula. In the second part, we will concentrate on the generalized Lambert series $\sum_{n=1}^{\infty} \frac{n^{N-2h}}{e^{n^N x}-1}$ studied by Kanemitsu, Tanigawa and Yoshimoto, which we recently found to be located on page 332 of Ramanujan's Lost Notebook in a slightly more general form. In a joint work with Bibekananda Maji, we have extended a transformation of this series given by Kanemitsu, Tanigawa and Yoshimoto. The novel feature of this extension is that it not only gives Ramanujan's formula for $\zeta(2m+1)$ but also its beautiful new generalization linking $\zeta(2m+1)$ and $\zeta(2Nm+1), N \in \mathbb{N}$. This extension gives, as special cases, several results on transcendence of certain values including a transcendence criterion for Euler's constant γ. We end the talk with some results from a very recent joint work with Bibekananda Maji, Rahul Kumar and Rajat Gupta on the series $\sum_{n=1}^{\infty} \frac{n^{N-2h} \exp(-an^N x)}{1-\exp(-n^N x)}, 0 < a \leq 1$.

Invited Talk 12.

Title: India's Crosscultural Exchanges in Mathematics and Astronomy

Speaker: MICHEL DANINO; IIT Gandhinagar, INDIA

Abstract: India has a long record of crosscultural exchanges in the fields of mathematics and astronomy. With fairly dim beginnings going back to classical Greece, this presentation will move on to China, the Arab world and medieval Europe. It will also trace the discovery of Indian science by European astronomers and mathematicians from the 17th century onward some of them enthusiastic, others critical and dismissive which, despite or thanks to the controversies, did end up laying the foundations for the academic discipline of history of Indian science.

Selected Talk 1.

Title: Relevance of Paramesvara's commentary on Lilavati

Speaker: ANIL NARAYANAN, N.; Chinmaya Sanskrit University, Kerala,INDIA

Abstract: The text *Lilavati* ranks top with respect to the number of commentaries. As is well known, Kerala has got a rich tradition on Mathematics. As many as eleven Keralite commentaries on *Lilavati* have been identified, five in Sanskrit and the rest in Malayalam. Keralite Sanskrit commentaries like *Buddhivilasini* and *Kriyakramakari* had critically edited and have been widely referred by the experts in the field. The present author has edited the commentary of Vatasseri Paramesvaran Namputiri on *Lilavati* and the paper pertains to this commentary of Paramesvara.

The paper discusses the critical apparatus used for editing the text. The features of the commentary like the style of writing, the abundance of mathematical examples, the charm of illustrations and supplementing optional/ extended *karanasutra*-s, etc. will be discussed and thereby the relevance of the commentary in the history of Kerala School of Mathematics will be evaluated.

Selected Talk 2.

Title: Novel mathematical modelling of *praṇakalāntara* by Mādhava

Speaker: ADITYA KOLACHANA; IIT Bombay & K. MAHESH; IIT Bombay

Abstract: Mādhava of Saṅamagrāma the illustrious founder of the Kerala School of Mathematics and Astronomy is credited with a number of important results, including infinite series for pi, as well as inverse tangent, sine and cosine functions, in addition to a number of astronomical innovations. While the works of his lineage of disciples have been studied to some extent, Mādhava's own works of which only a few survive remain poorly studied by modern scholars.

Mādhava's *Lagnaprakaraṇa* is an astronomical work dedicated to the determination of the *lagna* or the ascendant. The *prāṇakalāntara*, or the difference between the longitude and the corresponding right ascension, is an important astronomical parameter used in determining the ascendant, as well as the equation of time in Indian astronomy. In our presentation, we will highlight some of the novel algorithms described to calculate the *prāṇakalāntara* in the

hitherto unpublished manuscript *Lagnaprakaraṇa*, with a focus on the different geometric approaches adopted by Mādhava to arrive at the same result in a variety of ways.

Selected Talk 3.

Title: $\sin 18$ based on Bhaskaracharya's *Siddhanta Shiromani*

Speaker: ARADHANA ANAND; Delhi Public School, Bangalore South, INDIA

Abstract: This talk is about finding the value of sine 18 using a method based on Geometry and Quadratic equations. This result is described in Bhaskaracharyas Siddhanta Shiromani. The speaker then extends this method to find the value of sine 54 and also attempts to capture the result in verse form (in Sanskrit). The presentation is in Sanskrit (followed by an English translation).

Selected Talk 4.

Title: Contributions of lesser known mathematicians brought forth by Munīśvara

Speaker: K. MAHESH; IIT Bombay & ADITYA KOLACHANA; IIT Bombay

Abstract: The *Līlāvatī* of Bhāskarācārya (12th. cent. CE) is a popular text on Indian mathematics. Scores of commentaries and translations on it attests that it has been extensively used to introduce arithmetic and geometry in India for several centuries since its composition. The *Nisṛṣṭārthadūtī* (lit. the emissary of the bestowed meaning) is a brilliant, but as yet unpublished Sanskrit commentary on the *Līlāvatī*. Its author Munīśvara (1603 CE), the son of Raṅganātha, a resident of Vārāṇasī is known to have composed a few other works such as *Siddhāntasārvabhauma*, *Marīcivyākhyā* and *Pāṭīsāra*.

In his *Nisārthadūtī*, apart from explaining the rules and examples in the original text, the author makes useful and erudite discussions by way of providing demonstrations and proofs along with those provided by other mathematicians. Not only does he quote pioneers like Āryabhaṭa, Brahmagupta, Ṣrīpati, Āryabhaṭa II etc. in various places, but also throws light on lesser known scholars such as Lakṣmīdāsa, Rāmacandra, Sūrya Daivajña, Bālakṛṣṇa, Viṣṇu Daivajña, Keśava Daivajña etc. As a well versed critique, Munīśvara appropriately accepts or criticizes the views of these scholars. In our presentation, we would like to bring forth this aspect of Munīśvara.

Selected Talk 5.

Title: Using Lilavati to Create Interest in Mathematics: Some Experiences

Speaker: S. AGARKAR & R. GONDHALEKAR; VPM's Academy of International Education and Research, Thane, INDIA

Abstract: Mathematics is the subject of dislike for a majority of students in India. The studies show that this indifference is due to dull and dry style of mathematics teaching, non-relevance of problems to everyday life and underdeveloped learning prerequisites among the students. These problems need to be dealt with seriously to make mathematics popular among school students. The authors have been trying to tackle the issue by using problems from Lilavati, a mathematical treatise written by Bhaskaracharya in 12th century. They have been conducting workshops on Lilavati for the benefit of school students since June 2014. What began as a celebration of 900th birth anniversary programme of Bhaskaracharya has now become a regular activity of the Vidya Prasarak Mandal, Thane. More than 100 workshops, each lasting for three hours, have been conducted so far in three different states of India namely Maharashtra, Madhya Pradesh and Andhra Pradesh. Relevant problems are chosen from Lilavati and students are made to solve them individually during the workshops. This mode of interaction has been found beneficial in many ways. Since the problems in Lilavati are written in poetic style students get attracted to them. Direct connection of problems to day to day life motivate them to solve them. Reference to animals and plants in the problems prove to be an added attraction for school children. Girls, who usually keep away from mathematical tasks, are found to take great interest in solving problems from Lilavati. Conducting workshops based on Lilavati, thus, achieves gender equity apart from generating interest towards mathematics. The first hand experiences gained in conducting these workshops along with pedagogic implications for teaching school mathematics will be discussed during the presentation.

Selected Talk 6.

Title: Problems pertaining to the determination of direction, place, and time in Śrīpati's *Siddhāntaśekhara*

Speaker: JAMBUGAHAPITIYE DHAMMALOKA;
University of Canterbury, NEW ZEALAND & Department of Classical Languages, University of Peradeniya, Sri Lanka.

Abstract: Finding the cardinal directions at ones location is very important practices in Indian tradition as it plays a crucial role in the constructions of Vedic altars, individual houses, temples etc. Similarly, knowing the exact sunrise at the observers location is key to a variety

of activities social, religious, cultural etc. Hence, all astronomical texts discuss problems related to determination of direction, place, and time (*dik-deśa-kāla*). Śrīpati, an astronomer who flourished during the 11th cent. in Maharashtra, discusses these problems at length in the fourth chapter of his *Siddhāntaśekhara* called *Tripraśnādhikāra*. During our presentation, we will discuss the methods outlined by Śrīpati in his *Siddhāntaśekhara* .

Selected Talk 7.

Title: Soothram Sulabham Sundaram - A Sulbasutra method to find Rationals between Surds

Speaker: ARADHANA ANAND; Delhi Public School, Bangalore South, INDIA

Abstract: This talk is about a solution to a 10th Standard Mathematics problem of finding rational numbers between surds. The solution is based on a geometrical method from a very ancient text - the Baudhayana Sulbasutras. The presentation is in Sanskrit (followed by an English translation).

Selected Talk 8.

Title: Indian Mathematics in the mainstream

Speaker: PRIYAMVADA, N.; New Jersey, USA

Abstract: This paper presents the Indian sources of some of the most frequently studied mathematical results in schools and colleges around the world. Many of these results are now identified by the name of the mathematician, usually Western or Greek, who is considered to have first discovered it in the Western context. However, documented Indian primary sources do exist to prove that many of these results in algebra, geometry and combinatorics were first discovered by Indian mathematicians at a far earlier date than their Western counterparts. The paper will quote primary sources in Sanskrit, and analyze and interpret the mathematical language used. It will also shed some insight into the processes used by ancient Indian mathematicians to arrive at their results. In particular, the paper will discuss the Pythagorean Theorem, the quadratic equation, permutations and combinations, and the Fibonacci sequence. Too often, the mention of Sanskrit is associated in the popular imagination only with the language of prayer and ritual, and to some extent, with literature. The

information presented in this paper needs to be disseminated at the very least among Indian school and college students, to illustrate our common and extensive intellectual heritage.

Selected Talk 9.

Title: The Kerala Rope Technique Documenting the Practice of Altar Space layout Using Knotted Ropes, of Kerala Vedic Tradition

Speaker: SIVANANDAN D.S.; Centre for Spiritual Studies, Amritapuri Campus, Amrita University, INDIA

Abstract: In Vedic India the religious ceremonies and sacrifices were performed on altars and not in temples[2]. The altars are raised platforms made of bricks. The altar takes various shapes and dimensions, depending upon the purpose of sacrifices. The design and construction of these altars gave rise to a new branch of knowledge known as Śulbasūtras. The origin of the word can be traced to the words Śulba[3] (rope) and sūtra (aphorisms). In fact all the constructions was done using ropes and pegs.

The altar space in general consists of a pracinasāla , mahāvedi and uttaravedi. The pracinasāla includes four minor altar elements of various geometrical shapes like semi-circle, rectangle and square. The mahāvedi is an east facing trapezium to the east of pracinasāala. The mahāvedi also has spaces and elements of various geometric shapes and with specific relative position. Uttaravedi lies in the in the west of mahāvedi and can be in different shapes like falcon, tortoise and circle[4].

The entire altar space has complex layout and involves large number of measurements. In Kerala Vedic tradition, the entire altar space layout is marked in ground using knotted ropes and pegs. The instructions for such layout details are given in handbooks of Vedic priests.

In this paper an attempt has been made to document one such practice as mentioned in the handbook Mārancheri Bhāsa. This is a Malayalam commentary of Kārikās of Yogiyār[5] and was composed around 2nd half of 18th century. Mārancheri Bhāsa gives details such as various units of measurements, types of ropes, relative position of knots in the ropes and the technique of marking altar space using these knotted ropes. The current documentation

[2]Explaining the gods, R.L. Kashyap, Sri Aurobindo Kapali Sastry Institute of Vedic Culture, Bengaluru

[3]The root *śulb* signifies measuring or act of measurement

[4]Chapters 5 to 21 of Baudhāyana Śulbasūtra give the details of 14 types of uttaravedies.

[5]These are Sanskrit verses which gives the construction yāgasāla and the details of Agni ritual. There are three kārikās, (one each for square falcon, falcon with five set of nail and one with six set of nails) which explains the details regarding the three most common Agni of Kerala traditions. These kārikās are composed by an ascetic from Taikkat Mana of Covvaram grāmam. The period of the work is 16th c. CE as per Ulloor S Parameswaran.

is done using scale model of knotted ropes. In future there is a plan to document the entire technique using knotted ropes of original dimension.